So You're Going to Lead a Math Circle
(preliminary version)

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August 16, 2013

Abstract

You've been invited to lead a math circle and you've never done it before. This article will not only try to explain what a math circle is (as opposed to, say, a normal lecture, or a math club meeting), but it will include some hints about what to prepare, how to prepare, and what you can expect.

1 Math circles in general

1.1 What is a math circle?

A math circle is a group of students (usually motivated high school or middle school students) led by a mathematician who get together each week to learn mathematics. Often the leader changes regularly which has a few advantages:

- The leader doesn't get burned out; it's easy and fun to prepare a couple of presentations per year for motivated students.
- The students see different mathematical styles and different topics.
- The leader can make the same presentation at multiple circles if there is more than one circle in the area.

Math circles are different from the typical high school "math club":

- Circles emphasize problem solving.
- Circles don't necessarily cover material from the standard curriculum.
- Circles get students to think; they are generally not designed to drill the students for mastery of a skill or topic (although sometimes they can be designed to do this and the students don't even realize that it's happening).

To assure that the circle session you lead goes as well as possible:

- Circle sessions often concentrate on problem solving techniques applicable in many areas. Sample circle topics include: symmetry, the pigeon-hole principle, divisibility, counting, probability, invariants, graphs, induction, plane geometry, or inversion in a circle.
- Hand out a set of problems a week before your session. Not too many, perhaps three or four, but seductive. Include an easy one and a challenging one.
- Try not to lecture. Even though introducing new theory and techniques is an integral part of math circles, your sessions should be as interactive as possible. Score yourself: 1 point per minute you talk; 5 points per minute a student talks; 10 points per minute you argue with a student; 50 points per minute the students argue among themselves.
- Divide students into groups of 2-4 to solve problems. Have them present their own solutions.
- Be encouraging, even about wrong answers. Find something positive in any attempt, but don't be satisfied until there is a rigorous solution. Wrap up each problem by reviewing the key steps and techniques used.
- If the kids cannot answer your question immediately, don't just tell them the answer; let them think. If they're still stuck, give hints, not solutions.

1.2 What is problem solving?

> "In problem solving, as in street fighting, rules are for fools!"
> — Sanjoy Mahajan at TEDxCaltech, 2011

The Sanjoy Mahajan quote above emphasizes the fact that you can't get too bound up in rules for solving problems that you don't know how to do when you start. You've got to be flexible, and not get stuck. However, there are some strategies for helping to solve problems, and many of them are designed to help you get out of a fixed mindset. We will get to those later.

In the list at the end of the previous section, we said that circles emphasize problem solving. What is problem solving? The best definition we've seen of this is due to Paul Zeitz who describes it by defining the difference between a "problem" and an "exercise":

- An "exercise" is something you know how to do already, even though it may involve a long, ugly process. For example, multiplication by hand of two 10 digit numbers is an exercise. We have an algorithm that, provided we make no mistakes in our arithmetic, guarantees us the right answer. We know how to find the solution, even before we are given the two numbers. Although it may be hard to do, for anyone who learned to multiply, it is "just" an exercise.
- A "problem" is something you don't know how to do and don't have a formula for. Finding the maximum surface area of a tower with a height of 13 units built

exclusively from blocks with side lengths of 2^n is an example of a problem. It involves lots of arithmetic, but the solution is not immediately clear and there is no common formula.

- Following are a few more examples of problems. We will refer to these throughout the rest of this document. In what follows, we will use a notation like "[2]" or "[4]" to refer specifically to the second or fourth problems below, et cetera.

- Note that some exercises for experienced students would be problems for a beginner. If you know the formula for adding an arithmetic sequence then problem [1] below becomes just an exercise. The "problems" below are usually problems for middle school and high school kids.

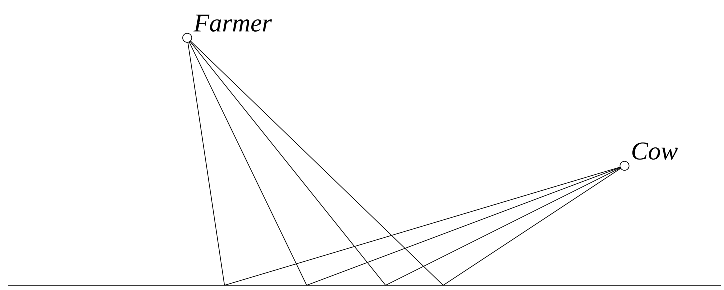

Figure 1: The Farmer and the Cow

1.3 Example problems

Partial solutions to these problems appear in Section 6.

1. Add all the numbers from 1 through 1000.

2. A farmer and a cow are on the same side of a straight-line river. (See Figure 1, where various possible routes for the farmer are illustrated. The river is the horizontal line.) The farmer has to walk to the river, get water in a bucket, and take it to the cow. What's his shortest path?

3. On a blackboard are written the numbers 1 through 100. At every stage, two are selected, erased from the board, and their sum plus product is added to the list on the board. At any stage, you're free to choose any two numbers. When the board is reduced to a single number, what possible values can it have?

4. The game of nim. There are two players and they begin with a pile containing 20 pennies. They alternate moves, and for each move, a player can remove 1, 2 or 3 pennies from the pile. When the pile is empty, the game is over and the player who cannot make a move loses. Does the first or second player win, assuming both use optimal strategies.[1]

[1]There is a great variation on this problem (and much more difficult and mathematically interesting)

1.4 Problem solving strategies

Here is a list (due to Joshua Zucker) of some of the more important strategies that can be used to approach a problem. These are not necessarily listed in order of importance.

1. **Do something**

   Don't just stare at a blank paper: "Mathematics must be written into the mind, not read into it. 'No head for mathematics' nearly always means 'Will not use a pencil.'" – Arthur Latham Baker.

2. **Patience**

   - Young kids don't have it.
   - Many students think that all math problems can be rapidly solved, and that idea is reinforced by the standard US school curriculum.
   - Many have the misconception that the interesting part of a problem is the solution, but sometimes the thought process required to get there is far more important.

3. **Special cases**

   - Get your hands dirty. For example play the game in [4] a few times.
   - Make and solve an easier problem. For [1], add much shorter lists by hand; for [3], start with a much shorter list; for [4] start with a smaller pile.

     One of the surprising differences between a great problem solver and a mediocre one is how they simplify problems like these. A mediocre solver might change the 1000 in problem [1] to 10. A good solver will change the 1000 to 1, then 2, then 3. It's almost always useful to look at the very smallest versions of a problem. And they're a lot easier to work out, usually.
   - Work out a specific example.

4. **Organization** If you're working on a problem that can be split into cases, make sure you've got all the cases. If you're doing experimental calculations, keep track of them in one place, et cetera.

   - Keep track of special cases in [1]: 1, $1+2$, $1+2+3$, $1+2+3+4$, and so on.
   - Here are a few other examples of counting problems where organization will be critical to finding the answers:

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which assumes that an animal shelter has some number of puppies and perhaps a different number of kittens. For your "move", you can adopt any number of puppies (and no kittens) or any number of kittens (and no puppies), or you can adopt an equal number of both. You and your opponent alternate moves, and the one who empties the shelter wins. Given the initial number of puppies and kittens in the shelter, what is the optimal strategy? This problem is also called "Wythoff's game".

- Count number of trees with n nodes. (A "tree" is a set of nodes connected by line segments so that there is exactly one path between any two nodes. In other words, there are no loops. This is a hard problem to solve completely, but making the list of the first few sets of examples is a good exercise in organization.)
- Given n lines in the plane, count possible numbers of points of intersection. There are lots of possibilities: if all lines are parallel, there are none; if the lines are in general position, there are a lot. (This problem is also difficult to solve in general, but again, the enumeration of the smaller cases is a good exercise.)
- Count the number of shortest paths through a 4×3 grid from the upper left corner to the lower right corner.
- Count number of pentominoes. A pentomino is like a domino, but made with 5 squares connected together. If you don't count mirror images, there are 12 distinct pentominoes.

5. **Look for a pattern** In problems $[1]$, $[3]$ and $[4]$, look for patterns in the easier problems where the "large" number is replaced by 1, 2, 3, 4, et cetera. If you can find a formula that seems to work for them, the form of the formula often provides a clue.

6. **Generalize**

   - Create a "knob": turn problem into a series of problems, as we did in problems $[1]$, $[3]$ and $[4]$ when we made the large number in the initial problem variable. But this is just one sort of "knob"; in $[3]$ we could change the operation from "add the sum and product of the numbers" to something simpler, like "add the numbers". Turn a 3-dimensional problem into two dimensions.
   - Use algebra: in $[3]$, replace "two numbers" by "x" and "y" and the operation "add the sum and the product" by $xy + x + y$.

7. **Symmetry** Remember that symmetry is not just geometric:

   - In $[1]$, we have $1 + \cdots + 1000 = 1000 + \cdots + 1$.
   - In $[2]$, flip the cow over the river.
   - In $[3]$, if you exchange x and y the result is the same: $xy + x + y = yx + y + x$.
   - In $[4]$, if we match the move n with the counter-move $4 - n$ we have a type of symmetry. Nim games become more difficult if the moves disallow symmetry. For example, in a nim game suppose that for each move you are allowed to take only 2, 5 or 7 pieces.

8. **Wishful thinking**

- Factor $x^4 + x^2 + 1$.

  This seems difficult, but it would be easy if you had $2x^2$ instead of x^2. So just change it to what you want (but correct it, too):

  $$x^4 + x^2 + 1 = x^4 + 2x^2 + 1 - x^2 = (x^2 + 1)^2 - x^2.$$

  But that's just the difference of two squares, so:

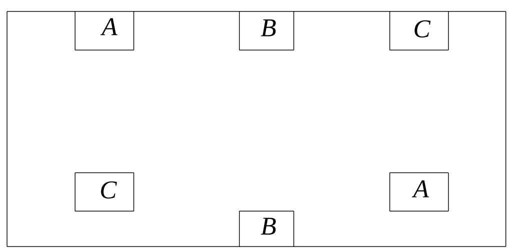
  $$x^4 + x^2 + 1 = (x^2 + 1 + x)(x^2 + 1 - x).$$

- In [3] the expression $xy+x+y$ is almost equal to $(x+1)(y+1)$, so change it in that way:

  $$xy + x + y = xy + x + y + 1 - 1 = (x + 1)(y + 1) - 1.$$

- Often in geometry you are working with an arbitrary triangle, and you would like to have a right triangle instead. It is always possible to drop an altitude from a vertex to divide the original triangle into two right triangles.

- In the following diagram, connect the corresponding boxes (A with A, B with B and C with C) using paths that do not cross each other or leave the surrounding box. The solution appears in Section 6.



9. **Working backwards** In problem [4], don't start from the beginning of the game; start from the end. If the game is nearly over and there are 1, 2 or 3 pennies, the person whose move it is can win immediately by taking all of them. If there are 4, then whatever move is made leaves 1, 2 or 3, so a person facing a pile of 4 pennies will lose if his opponent plays well. Et cetera.

10. **Invariants, Monovariants** Sometimes you can find a quantity that is unchanged after every operation (called an invariant) and sometimes you can find a quantity that changes only in one direction (a monovariant). Here are a couple of examples:

    - In [3] the process must end because each time the operation is performed, there is one fewer number on the board. So the number of numbers on the board is a monovariant. In fact, since it is reduced by one each time, the process will be complete in exactly 99 moves (starting with the numbers

1 through 100 on the board initially). There's another invariant here, too, although it's hard to see until you solve the problem. If you add 1 to every number on the board and multiply all those together, the result is the same after any two numbers are combined.

- Paul Zeitz proposed the chocolate-bar problem: you begin with a chocolate bar that has 8 squares by 4 squares. Your move is to take a chunk and snap it along a line (vertical or horizontal) between squares. You alternate making moves with your opponent, and the winner is the last person to be able to break a piece. The monovariant in this problem is that each time there is one more piece and the game has to be over when there are $8 \times 4 = 32$ pieces.
- To prove Euler's formula relating the number of faces, edges and vertices of a convex polyhedron, you are basically trying to prove that the number $F - E + V$ is invariant (and in fact, always equal to 2).[2]

11. **Recycle** If you've worked out values for simpler versions of the problem, perhaps you can use them to work out harder versions.

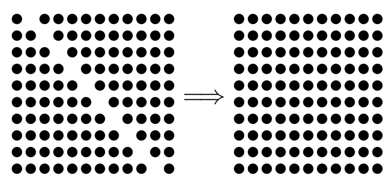
    - In [3], once you've convinced yourself that the order in which the numbers are combined makes no difference, if you've got the final answer for a board beginning with $1, 2$ and 3, then to work out the answer for the board beginning with $1, 2, 3$ and 4, simply combine the previous result with the number 4.
    - In [4], once you've worked out who the winner is in games beginning with penny counts of $1, 2, 3, \ldots, k$, it's easy to work out the winner for a game beginning with $k + 1$ pennies.
    - If you're trying to count pentominoes (5-square "dominoes") and you've already counted the monominoes, dominoes, trominoes and tetrominoes (which you should have), you can just look at ways to add a square to all the tetrominoes you've got to get a complete list of pentominoes.

12. **Give things names** In [3], name the operation. Suppose you call it "$*$", then you would define it as:
$$x * y = xy + x + y.$$
As you work with the operation, you'd like to prove:
$$\begin{aligned} x * y &= y * x \\ x * (y * z) &= (x * y) * z, \end{aligned}$$
but be sure to give *those* properties names, too: the commutative and associative (respectively) laws for the operation $*$.

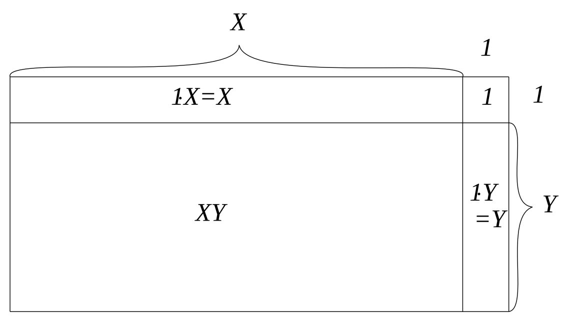
[2] At: `http://www.geometer.org/mathcircles/euler.pdf` you can find notes for leading a circle based on Euler's formula.

13. **Make a picture** It is important to teach students how to create (and think about) visual representations of a problem. This idea may be abstract for young students (good example: hand-shake problem where nodes represent people and lines represent handshakes).

    - It's obvious you need a picture for [2].
    - Draw a triangle of dots for [1]. Note that if you double it and flip it over, you will have a rectangle of dots that are easily counted. Half that number is your answer.

    - For problem [3] the following picture may be useful:

      The area of the entire rectangle is obviously $(x+1)(y+1)$, but you can also see that $xy+x+y$ is the area of the whole rectangle with the exception of the little 1×1 square in the upper-right corner.

14. **Look for extremes** Sometimes when you're trying to understand a problem, it's useful to look at extreme situations.

    Suppose there are a million points in a circle. Prove that there is a line that puts exactly half of them on one side and half on the other.

    Imagine drawing all the lines connecting all the pairs of points. Find a point P outside the circle that is not on any of those lines. Draw a line through P that does not go through the circle, so *all* the points are on one side of it (an extreme line). Sweep that line through the circle. Since P is not on any line connecting two points inside the circle, it will cross one point at a time as it is swept. After crossing a half-million points, you will have the desired line.

    For problem [2], one extreme case would be to have either the farmer or the cow or both right at the river. It's also clear from this that if the cow is very close to the river, the shortest path would touch the river very close to the cow.

2 Math circles in particular

The authors have extensive experience with a number of different math circles:

- Main campus (SFSU) and two satellites (Mission HS and Thurgood Marshall)
- San Jose State University Math Circle
- Berkeley Math Circle
- Stanford University Math Circle

2.1 Variations in age, ability, motivation

Within a circle the variables of greatest significance will be ability and motivation. Though the SFSU circles are categorized by age level, they are really tiered according to ability. The idea is, if a student is in your circle then he/she can handle it (and should not be a behavior problem despite any age differences).

Expect great differences in ability, even among students of similar age.

Motivation may or may not be a problem, depending on your students, but different kinds of activities are more conducive to (i) different types of learners, (ii) different age groups, and (iii) different maturity levels.

2.2 What to expect

Things won't go exactly as you predict. Prepare 2 to 3 times as much material as you think you could possibly go through. Be prepared to use all or very little of it. Actually, in practice, people tend to prepare far too much material to cover in an hour (or two hours).

Be prepared to go faster or slower. Predicting the appropriate pace can be the most difficult part of a lesson.

Don't worry if you don't "complete" the topic or lesson. This isn't regular school. There are no curriculum objectives and you may not be explicitly training our students for math competitions. Let a lesson or topic go where it may.

It's OK to go off on a good tangent. A wonderful example arises from the farmer/cow problem [2]. Suppose for a fixed farmer and cow you look at all the rivers that would cause the farmer to go the same distance. These will "obviously" form an envelope of lines of an ellipse with the farmer and cow as foci. Since in the standard solution to the problem, it's clear that the angle of the line approaching and leaving the river is the same, this shows that light or sound emanating from one focus of an ellipse will be refocused at the other focus, and will all arrive at the same time.

Another example with the farmer/cow problem arises if the farmer goes slower carrying a full bucket of water and the goal is to get him to the cow as quickly as possible. This leads to Snell's Law of refraction in physics.

2.3 What to do when problems arise

- Kids who are lost.
  - Go to the strategy board. Go through the list.
  - Pair him/her with a stronger student. Explain that they need to work as a team.
  - Group students who are struggling together and go over the problem again with them (as a group), addressing any difficulties. You may want to give them a simpler (version of the) problem.
  - Always have good hints ready. Ask for good hints from the class (starting points or ways other students have thought about the problem).
  - Have simpler problems ready or ways to incorporate slower students into a complex problem (keep track of what's been done, record the data, etc.)
  - Have struggling students explain the problem back to you. See if you can determine where he/she got lost.
- Kids who are bored.
  - First, try to engage them. Maybe they don't understand the problem. It is too hard/complex or is it too easy? If it's too easy, make sure they understand the process and can explain it. Right answers are not enough.
  - If students choose not to participate because they think it's boring, feel free to tell them that may sit quietly if they choose not to work, but may not disrupt the rest of the class.
  - Give them more problems and harder ones.
  - Ask them what they know. What have they discovered? They might just need attention. Challenge them! "Oh, I see you have figured out this, but what about this possibility?"
- Kids who don't want to be there.

  We only have two rules at Math Circle: (1) that you have fun doing math and (2) that you want to be here. You should tell students this. Let them know that if they don't want to be there, then they shouldn't be there and that's okay. Tell them we are happy to have them in our class if they wish to participate and think they can have even a little bit of fun doing math.

  If it is clear that a student is there by no choice of his/her own, you may want to ask to speak with the adult that he/she came with. This is a voluntary program, students who wish not to be there take away from the classroom environment and spirit of math circles.
- Kids who are disruptive (too active, usually)
  - Usually they are bored. Have more problems for them.

– Give them something to do: tasks (passing stuff out, helping other students), challenge them!
– Give them an opportunity to speak/share ideas with the class, but call on them, so that they know you are in control.
– Have them sit alone.
– Speak to the student individually after class. Calling them by name to inform him/her that you wish him/her to stay has an impact all of its own.
– It's not ok for kids to be disruptive, they need to know this.
– You can have students sit outside the classroom quietly if they simply cannot behave.
– You can also send them to an older circle if you think they are bored.
– Or you can send them to a younger circle if they are behaving like a younger student.

3 Math circles dissected

Here are some features of topics that make them especially suitable for a math circle:

- interactive problems
- open problems (something for everyone)
- easy to explain problems
- interesting problems
- problems with manipulatives
- problems that encourage teamwork
- problems that encourage experimentation
- problems that trick kids into doing "drills"

4 Sessions and Problems

In June, 2011, a workshop was held at the American Institute of Mathematics (AIM) in Palo Alto with the topic, "How to Start a Teacher's Circle." A teacher's circle is very similar to math circles for kids that are the topic of this article, except that the participants are middle-school teachers. Toward the end of the workshop, we had a brainstorming session where we wanted to make lists of features that help to make a good circle and that help to make a good problem. Not all of the ideas listed necessarily apply to a standard math circle, but the lists were quite good, so I am including both of them in this section. One of the participants, when asked what makes a good problem, said, "It's like pornography; I don't know exactly how to describe it, but I know it when I see it!"

In succeeding years, similar workshops of the same sort were held and this exercise was repeated, so the list below actually represents a combination of the ideas from all the workshops. Some of the suggestions may seem somewhat contradictory, but there certainly are sessions where one or the other is appropriate.

Anyway, here are the lists:

Criteria for a good session:

- Excitement
- Hands on
- Clarity
- Make everyone feel smart/important
- Appropriate facilities/environment
- People show up
- Good leader $\rightarrow$ less talk
- Leader is well-prepared
- Professional versus intimidating environment
- Respect
- Happy ending
- Careful choice of language (avoid technical terms, or expressions like "we all know")
- Food and drink
- Fun
- Team building
- Handouts (no need for note taking)
- Debug/debrief time
- Want to come back
- Collaborative $\rightarrow$ people at tables
- Involve everybody
- Useful take-away
- Enriched as mathematician
- Time to think
- Group structure
- OK not to know
- Effort/engagement
- Pacing $\rightarrow$ time to think
- Moments of discomfort/challenge
- Usable in classroom
- Not lecture style
- Respect skill/experience of all
- Soft entry
- Less is more: one or two big problems
- Community
- Modeling problem solving and instruction
- Invite engagement/interaction
- Some parts finish; some for later
- Safety (emotional/intellectual)
- Probing, guiding questions

- Enthusiastic participants
- Enough background material
- Closure
- Debriefing time
- Participants at board
- Participants make up the questions
- No lecturing
- Use of manipulatives → hands-on
- Multiple representations
- Guided, but not too direct

Criteria for a good problem:

- Simple statement
- Easy introduction
- Easy to begin = easy entry
- Accessible for everyone
- Sensory input
- Multiple approaches
- Use different abilities
- Can be appreciated on many levels, like a good novel
- Leads toward more complicated problems
- Not necessarily immediately applicable to a classroom
- Open-ended
- Engaging
- Steady/smooth progression of challenge
- Novelty
- Simplicity (no computers/calculators required)
- Generates other questions
- Can be replicated
- Deeper prospects
- Invites collaboration
- Leader is enthusiastic
- Has hidden structure
- Clear vocabulary
- Puzzles
- Connections
- Many important breakthroughs
- Applications
- Manipulatives when possible
- Solution and approach not obvious
- Standards-based
- Adaptable to different levels
- Resolution, but with more to follow
- Diverse connections
- Reach some conclusion/climax

- Aha! moments
- Oops! moments
- Struggle $\to$ empathy
- Setting rather than question
- Ill-defined
- Surprising results/connections
- Multiple knobs
- Tip of the iceberg
- Uses grid paper
- Unsolved twist

5 Useful Materials

Three books worth having are:

"Mathematical Circles (Russian Experience)", by Fomin, Genkin, and Itenberg, American Mathematical Society, 1993.

"Circle in a Box", by Sam Vandervelde, MSRI Mathematical Circles Library, 2009.

"A Decade of the Berkeley Math Circle: The American Experience, Volume 1", Zvezdelina Stankova, Tom Rike (editors), MSRI Mathematical Circles Library, 2008.

Here are a couple of useful websites:

`http://www.mathcircles.org/` This is a central site with pointers to many, many individual circle sites around the country and lots of additional information.

`http://www.mathteacherscircle.org/` Teachers' circles are almost the same as math circles, but the "students" are middle school teachers. In addition to math topics, these circles also include discussions of pedagogy. This site is like the one above, except that it is for teachers' circles.

`http://www.geometer.org/mathcircles` This is the personal circle site of one of the authors (Tom Davis) and it includes fairly detailed handouts for all the circles he has ever led. If you found this document online, you probably found it at this site.

6 Solutions to Problems

1. One simple solution is to note that $1 + \cdots + 1000 = 1000 + \cdots + 1$. If the unknown sum is S, then

$$\begin{aligned} S &= \quad 1 + \cdots + 1000 \\ S &= 1000 + \cdots + 1 \end{aligned}$$

   Add them together and we obtain:

$$2S = 1001 + \cdots + 1001,$$

   where there are 1000 copies of 1001. Thus $2S = 1000 \times 1001 = 1001000$, so $S = 500500$.

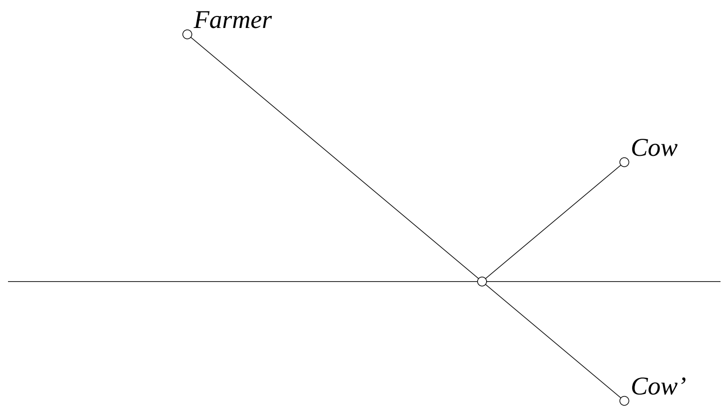


Figure 2: Solution: The Farmer and the Cow

2. Imagine reflecting the cow across the line that represents the edge of the river. (See Figure 2, where the position of the reflected cow is labeled "Cow'".) It's the same distance for the farmer to go to the original cow as the reflected cow, and the distance to the reflected cow is a straight line. Thus it's easy to construct the solution, and to see that the path of the farmer makes equal angles of incidence and reflection off the river edge, just like light bouncing off a mirror.

3. This takes a few steps, but if we call the operation of combining two numbers $*$, then we have:

   $$x * y = xy + x + y.$$

   A little algebra shows that $*$ is commutative and associative:

   $$\begin{aligned} x * y &= y * x \\ (x * y) * z &= x * (y * z) \end{aligned}$$

   Since it's commutative and associative, it's easy to see that the order in which the numbers are combined makes no difference.

   We also note that $x * y = (x + 1)(y + 1) - 1$. Looking at a few cases shows us that the following pattern holds:

   $$\begin{aligned} x * y &= (x + 1)(y + 1) - 1 \\ x * y * z &= (x + 1)(y + 1)(z + 1) - 1 \\ x * y * z * w &= (x + 1)(y + 1)(z + 1)(w + 1) - 1 \\ \cdots &= \cdots \end{aligned}$$

   Thus the value for the complete set of numbers from 1 to 100 is:

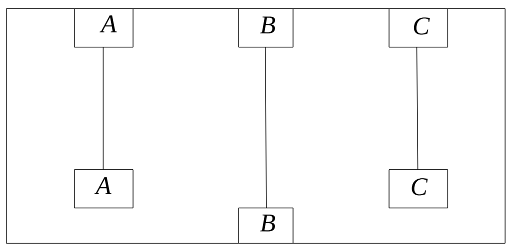
   $$(2)(3)(4) \cdots (101) - 1 = 101! - 1.$$

   A complete guide to using this problem in a math circle can be found here:

   `http://www.geometer.org/mathcircles/numbercombine.pdf`

4. This game is best analyzed backwards. For a given number of initial coins, we can decide whether the position is won or lost by the person forced to make a move when the pile has that number. A pile containing zero is clearly a loss. Piles with $1, 2$ or 3 are wins since the player can reduce any of those to a loss (zero) for his opponent. A pile of 4 is a loss since all the possible moves leave a win for the opponent. Piles with 5, 6 and 7 are wins, since all can be reduced to 4 which is a loss for the opponent. Continue the argument and it's easy to see that all piles that contain a multiple of 4 coins are losses and all the others are wins for the player who has to make a move.

5. (Connecting the boxes). To solve this problem by wishful thinking, the obvious thing to wish for is that the A and C boxes on the bottom were reversed. Then the solution would be trivial:



Now imagine that the lines are rubber bands, and just slide the C block to where it needs to go and then slide the A block to where it needs to go, in all cases just "pushing" the BB line ahead of the moving blocks. The final result is illustrated below: